RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [2016-19]

B.A./B.Sc. FIRST SEMESTER (July – December) 2016

Mid-Semester Examination, September 2016

Date : 10/09/2016 **PHYSICS (Honours)**

Time : 11 am – 1 pm Paper : I Full Marks : 50

[Use a separate Answer Book for each group]

(Answer five questions taking atleast one from each group)

Group – A

1. a) What are the physical significances of gradient? [2]
   b) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, –2, –1). [4]
   c) Find the curl of $(\vec{r}\, f(r))$ where f(r) is differentiable. [2]
   d) Prove $\vec{\nabla}\cdot(\phi\vec{A}) = (\vec{\nabla}\phi)\cdot\vec{A} + \phi(\vec{\nabla}\cdot\vec{A})$. [2]

2. a) Define a conservative field. [2]
   b) Write integral form of curl. [1]
   c) i) Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative field [2]
   ii) Find the scalar potential. [2]
   iii) Find the work done in moving an object in this field form (1,–2, 1) to (3, 2, 4). [1]
   d) Verify green's theorem for the closed region bounded by the curves y = x and $y = x^2$. [2]

Group – B

3. a) A particle moves with constant speed v around a circle of radius b. Find its velocity vector in polar coordinates using an origin lying on the circle. [2]
   b) The trajectory of a particle is defined by $r = r_0e^{\beta t}, \theta = \omega t$, where r_0, β, ω are constants. Show that for $\beta = \pm\omega$, the radial acceleration is zero, even though the radical velocity is increasing with time. Explain. [3]
   c) Three rail cars each of mass M is pulled with force F by an engine. Use suitable force diagram to find the forces on each car. [3]
   d) In an inertial frames, a projectile is fired with initial speed u in a direction θ w.r.t the x-axis. Let H and R be the maximum height and range in frame S. A frame S' moves with uniform velocity V along to x-axis, relative to S. Find the corresponding values H' and R' in S'. [2]

4. a) A particle is projected vertically upwards with an initial speed u in a medium that offers resistance kv^2 per unit mass where v is the instantaneous speed. Set up the equation of motion and find the maximum height reached. [4]
   b) Set up the differential equation of motion of a body of variable mass, moving in an applied force field $\vec{F}$, using the impulse momentum theorem. Hence, obtain the equation of motion of a rocket. Explain the physical significance of each term. [6]

Group – C

5. a) Show that if A and B are matrices which don't commute, then $e^{A+B} \neq e^Ae^B$, but if they commute then the relation holds. [4]

b) Find the eigenvalues and eigenvectors of the matrix $H = \begin{pmatrix} -2 & 3+4i \\ 3-4i & -2 \end{pmatrix}$. Write a unitary matrix U which diagonalize H by a similarity transformation, and show that $U^{-1}HU$ is the diagonal matrix of eigenvalues. [6]

6. a) Solve the non-homogeneous differential equation : $\frac{d^2y}{dx^2} + 4y = x^2 \sin 2x$. [4]

b) Solve the system of equations :

$\frac{dx}{dt} + 2x - 3y = t$; $\frac{dy}{dt} - 3x + 2y = e^{2t}$. [6]

Group – D

7. a) Determine the system matrix for a convex spherical surface separating two media and deduce the relation $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$ where the symbols have their usual meaning. [5]

b) Using Fermat's principle deduce the law of reflection for a concave spherical surface. [3]

c) Show that for an elliptic mirror the two foci are the two aplanatic points. [2]

8. a) Find the cardinal points of two thin lenses separated by a distance in air. [4]

b) Show that for a planoconvex or a planoconcave lens one of the principal points will always lie on the curved surface. [3]

c) Find the cardinal points of a waterdrop having $n = \frac{4}{3}$ and radius 1 mm when placed in air. [3]

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